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Thermal Transport Properties of Layered Materials: Identification by a New Numerical Algorithm for Transient Measurements[1]

Regine Model[2]

Transient methods are widely used to determine thermal transport properties. In some situations they can be used for homogeneous media to measure several properties either simultaneously or separately. In this context an analytic model is available and a well-posed inverse problem of parameter identification has to be solved. The examination of composite media is more complicated. The algorithm proposed here allows simultaneous determination of the thermal conductivity and thermal diffusivity of layered dielectrics by transient measurements. It is based on a plane source that acts both as a resistive heater and temperature sensor. For the technique to be successful two essential aspects have to be considered: firstly, the mathematical modeling of the measured data (the forward problem) and secondly, the problem of ill-posedness of the inverse problem. For the proposed measurement configuration, a new fast data analysis algorithm based on an analytic solution for the forward problem is presented. In principle, a numerical solution such as an FEM solution of the heat conduction equation can be used instead of the analytical one, but the computational effort is much greater. The inverse problem is formulated as an output-least-squares problem, which leads to a transcendent algebraic system of equations. The method was successfully tested for different situations.

KEY WORDS: heat conduction; inverse problem; multi-layered material; transient method; thermal conductivity; thermal diffusivity; thermal transport properties.

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1. INTRODUCTION

Information on thermal transport properties has increasingly gained in importance in the fields of engineering which try to reduce the energy involved, e.g., in process engineering and in the building industry. In the case of homogeneous media, besides classical steady-state methods, alternative transient techniques are now becoming widely used worldwide for all types of material [1–5]. The thermal conductivity λ and the thermal diffusivity a are derived quantities and, thus, cannot be measured directly. They rather have to be determined from related quantities, e.g., a temperature profile. In general, a heat flow of known rate, Φ, is passed through the material under test and the associated temperature profile $T(\boldsymbol{x}, t)$ is measured depending on the thermal properties. In several situations, these methods can be used to measure several properties either simultaneously or separately. In this context a well-posed inverse problem of parameter identification has to be solved.

The examination of composite media has only recently been considered with only a few results; [6–13]. Analytic approximations of the solution of the forward problem, which means simulation of the measuring signal, are available for homogeneous media, which in general lose their validity for layered composites. An alternative involves the application of numerical methods as finite-element and finite-difference methods but at the expense of considerable computational effort [7–10]. In some cases the experimental setup allows one-dimensional modeling of heat conduction in composite materials for which analytic solutions are derivable via Laplace transformation, e.g., Refs. 6, 11, 13, or a neural network method is used [12].

For the measurement configuration proposed here, a new fast data analysis is derived on the basis of an analytic solution of the heat conduction equation with piecewise-constant thermal transport properties corresponding to the separate layers. Here the Green's functions for the time-dependent case are used. The method allows the thermal conductivity and thermal diffusivity of layered dielectrics to be simultaneously determined by transient measurements. A plane heat source consisting of thin metal foil acts both as a resistive heater and as a temperature sensor.

In the first part of Section 2, the mathematical model for the transient temperature distribution applied to the proposed method is given. An analytic expression describing the measuring signal based on the Green's function formulation is derived in the second part. The section ends with the output-least-squares algorithm for the inverse problem, which leads to a transcendent algebraic system of equations.

In Section 3, the theoretical ideas of Section 2 are successfully applied to a layered sample where the algorithm is split into two steps. In the first step, the properties of the inner core and in the second step, those of the outer layer are identified. A summary is given in Section 4.

2. THEORY

2.1. Mathematical Model

The data analysis of the implicit measuring method consists of two parts: the first part is the so-called forward problem for which a mathematical model is derived relating the measured data to the thermal properties; this means that for known thermal properties of the sample and a known experimental setup, the corresponding measuring signal can be simulated. The second part is the inverse problem; for a given measuring signal and a known experimental setup, the thermal transport properties have to be identified.

The principle of the proposed method is shown in Fig. 1. A current-carrying metallic foil is clamped between two layered sample halves and simultaneously acts as a temperature sensor. This method works similar to the hot-strip technique; both methods are based on a step-wise heat source which is combined with the temperature sensor. The difference consists in the heat source geometry, on the one hand a plane source and on the other a hot strip. In another conceivable version, the temperature response

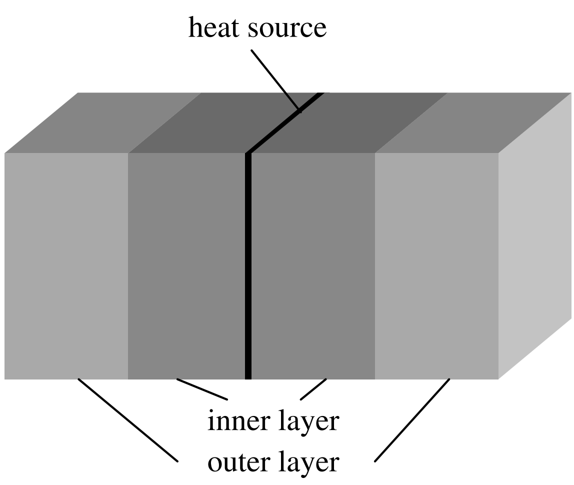

Fig. 1. Schematic diagram of the specimen for the proposed method.

could be measured by a separate sensor placed a distance h from the heat source but within the inner layer. It describes a step-wise transient technique based on a separated heat source and temperature sensor.

The choice of a plane source geometry provides the possibility of modeling the heat transfer process as a one-dimensional problem. Even for the case of multi-layered composites, an analytic solution of the corresponding partial differential equation for heat transfer can be derived as a solution of the forward problem. The length and width of the foil have to be sufficiently large for the heat losses at the surface to be negligible. Another way of allowing one-dimensional modeling of the temperature response may be realized by total insulation at the surface, i.e., the assumption of a homogeneous boundary condition of the second kind.

The one-dimensional formulation of the transient heat conduction problem for an m-layered slab is given as follows. The interfaces between the layers are located at $x = x_i$, $i = 1, 2, \ldots, m-1$ and the outer boundary surfaces at x_0 and x_m. Let λ_i be the constant thermal conductivity and a_i the constant thermal diffusivity of the i-th layer, $x_{i-1} < x \leqslant x_i$, $i = 1, \ldots, m$. We get the differential equation,

$$\frac{\partial T(x,t)}{\partial t} = \mathrm{div}(a \ \mathbf{grad} \ T(x,t)) + \frac{a}{\lambda} q(x,t) \quad \text{in } x_0 \leqslant x \leqslant x_m, \quad t > 0. \quad (1)$$

The heat source q is restricted to the thin foil and the thermal diffusivity a and the thermal conductivity λ depend on the diffusivities and conductivities of the single layers;

$$a(x) = a_i, \quad \lambda(x) = \lambda_i, \quad x_{i-1} < x \leqslant x_i.$$

The initial temperature at $t = 0$,

$$T(x,0) = T_0(x), \quad x_0 \leqslant x \leqslant x_m \quad (2)$$

can be different for each layer and also vary within a layer. At the outer boundaries we write the general form,

$$-\lambda_1 \frac{\partial T(x_0,t)}{\partial x} = h_0(T_1(t) - T(x_0,t)), \quad t > 0 \quad (3a)$$

$$\lambda_m \frac{\partial T(x_m,t)}{\partial x} = h_m(T_m(t) - T(x_m,t)), \quad t > 0, \quad (3b)$$

where $T_1(t)$ and $T_m(t)$ represent the—possibly time-dependent—ambient temperature of the first and the m-th (last) layer, respectively. Additional boundary conditions at the layer interfaces have to be satisfied,

$$\lambda_i T_x(x_i - 0, t) = \lambda_{i+1} T_x(x_i + 0, t) \quad (3c)$$

ensuring the continuity of heat flux at the interfaces. In the case of perfect thermal contact between the layers, we have the continuity of temperature,

$$T(x_i - 0, t) = T(x_i + 0, t), \quad i = 1, 2, \ldots, m - 1, \tag{3d}$$

and for the case where there is thermal contact conductance at the interfaces,

$$-\lambda_i T_x(x_i, t) = h_i(T(x_i - 0, t) - T(x_i + 0, t)), \quad i = 1, 2, \ldots, m - 1, \tag{3d*}$$

with the interface thermal contact conductance h_i at the i-th interface.

Let us adapt the general mathematical model to our special experimental situation. Due to the symmetrical setup of a centered inner source, the integration domain of Eq. (1) can be reduced to one half. Furthermore, concentrating on a sample simply consisting of two materials with perfect thermal contact, we get the following conditions for our specialized problem with $m = 2$ and $x_0 = 0$ corresponding to Eqs. (3a–3d)

$$-\lambda_1 \frac{\partial T(0, t)}{\partial x} = 0, \quad t > 0, \tag{4a}$$

because of symmetry the heat flux vanishes,

$$\lambda_2 \frac{\partial T(x_2, t)}{\partial x} = h_2(T_2(t) - T(x_2, t)), \quad t > 0, \tag{4b}$$

the outer boundary condition and the conditions at the interface,

$$\lambda_1 T_x(x_1 - 0, t) = \lambda_2 T_x(x_1 + 0, t), \tag{4c}$$

$$T(x_1 - 0, t) = T(x_1 + 0, t). \tag{4d}$$

If d is one half the thickness of the foil, we get the following form for the heat source q:

$$q(x, t) = q(x) = \begin{cases} q_0, & x \leqslant d \ll x_1 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

In reality, the materials and thus the thermal properties of the thin foil (10–20 μm) and the inner layer are highly different. Considering the foil as a separate layer or neglecting the separate layer leads to a small difference in the calculated temperature distribution. Nevertheless, this difference is covered by the measurement uncertainty as shown in Ref. 14

by finite-element simulations. For simplicity, the different material properties of the foil are neglected in the following. Then, the mathematical formulation is given by the heat conduction equation,

$$\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(a\frac{\partial T(x,t)}{\partial x}\right) + \frac{a}{\lambda}q(x) \quad \text{in } 0 \leqslant x \leqslant x_2, \ t > 0 \tag{6}$$

with the initial condition (Eq. (2)), the boundary conditions (Eqs. (4a, b)), the interface conditions (Eqs. 4c, d), and the heat source $q(x)$ given in Eq. (5).

2.2. Analytic Solution of the Forward Problem

To solve the nonhomogeneous heat transfer problem (Eq. (6)) in a layered composite medium, we start with a homogeneous problem with no heat generation,

$$\frac{\partial \overline{T}(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(a\frac{\partial \overline{T}(x,t)}{\partial x}\right) \quad \text{and} \quad \overline{T}(x,0) = T_0(x) \quad \text{for } 0 \leqslant x \leqslant x_2, \ t > 0 \tag{7}$$

to obtain the Green's function. When this function is available, the temperature distribution of Eq. (6) can be represented only in terms of the Green's function.

Assume a separation of variables in space- and time-dependent functions in the form,

$$\overline{T}(x,t) = u(x)\Gamma(t). \tag{8}$$

The time-variable function $\Gamma(t)$ is the solution of

$$\frac{d\Gamma(t)}{dt} + \beta_n^2 \Gamma(t) = 0 \quad \text{for } t > 0$$

given by

$$\Gamma_n(t) = e^{-\beta_n^2 t}. \tag{9}$$

The β_n^2 are choosen nonnegative to ensure the finiteness of temperature for $t \to \infty$. For homogenous boundary conditions of the second kind at the sample surface, zero is also an eigenvalue. Otherwise all eigenvalues are positive. The corresponding eigenvalue problem is given by

$$\frac{d^2 u_i}{dx^2} + \frac{\beta_n^2}{a_i} u_i = 0, \tag{10}$$

with the dimensionsless split eigenfunctions,

$$u_n(x) = \begin{cases} u_{1n}(x), & \text{for } 0 \leqslant x \leqslant x_1 \\ u_{2n}(x), & \text{for } x_1 < x \leqslant x_2 \end{cases}$$

where the index n indicates the dependence on the eigenvalue β_n. The solution is subject to the boundary conditions,

$$-\lambda_1 \frac{du_{1n}(x)}{dx} = 0 \qquad \text{at } x = 0 \tag{11a}$$

$$u_{1n}(x) = u_{2n}(x) \qquad \text{at } x = x_1 \tag{11b}$$

$$\lambda_1 \frac{du_{1n}}{dx} = \lambda_2 \frac{du_{2n}}{dx} \qquad \text{at } x = x_1 \tag{11c}$$

$$\lambda_2 \frac{du_{2n}}{dx} = 0 \qquad \text{at } x = x_2 \tag{11d}$$

where, for simplicity, we assume adiabatic conditions at the sample surface, that means that the surface conductance vanishes, $h_1 = h_2 = 0$. For nonvanishing surface conductance, the derivation of the analytic solution can be analogously performed. The general solution u_{in} of the eigenvalue problem (Eq. (10)) for a slab geometry can be written in the form,

$$u_{1n}(x) = A_{1n} \sin\left(\frac{\beta_n}{\sqrt{a_1}} x\right) + B_{1n} \cos\left(\frac{\beta_n}{\sqrt{a_1}} x\right) \qquad \text{for } 0 \leqslant x \leqslant x_1 \tag{12a}$$

$$u_{2n}(x) = A_{2n} \sin\left(\frac{\beta_n}{\sqrt{a_2}} x\right) + B_{2n} \cos\left(\frac{\beta_n}{\sqrt{a_2}} x\right) \qquad \text{for } x_1 < x \leqslant x_2. \tag{12b}$$

The first boundary condition (Eq. (11a)) requires that $A_{1n} = 0$. Without loss of generality, one of the nonvanishing coefficients can be set to unity since one coefficient is arbitrary. We have chosen $B_{1n} = 1$. Moreover, the solution (Eq. (12)) has to fulfill the remaining conditions (Eqs. (11b) to (11d)) yielding the following equation in matrix form for the determination of the coefficients A_{2n} and B_{2n}:

$$\begin{bmatrix} \cos\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right) & -\sin\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) & -\cos\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) \\ -\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}} \sin\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right) & -\cos\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) & \sin\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) \\ 0 & \cos\left(\frac{x_2\beta_n}{\sqrt{a_2}}\right) & -\sin\left(\frac{x_2\beta_n}{\sqrt{a_2}}\right) \end{bmatrix} \begin{bmatrix} 1 \\ A_{2n} \\ B_{2n} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{13}$$

The matrix of coefficients in Eq. (13) depends on the eigenvalues β_n of the problem (Eq. (10)), unkown so far. Nevertheless, they are determined by the requirement of the vanishing determinant, the condition for the existence of a solution of Eq. (13). After having determined the β_n, in general by numerical methods, we obtain for A_{2n} and B_{2n},

$$A_{2n} = \cos\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right)\sin\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) - \frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\sin\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right)\cos\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right), \quad (14a)$$

$$B_{2n} = \frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\sin\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right)\sin\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right) + \cos\left(\frac{x_1\beta_n}{\sqrt{a_1}}\right)\cos\left(\frac{x_1\beta_n}{\sqrt{a_2}}\right). \quad (14b)$$

Now, with Eq. (14) and the knowledge of the eigenvalues β_n, the eigenfunctions u_{in} defined in Eq. (12) are known and the general solution of Eq. (7) becomes

$$\overline{T}_i(x,t) = \sum_{j=1}^{2} \int_{x_{j-1}}^{x_j} \left[\sum_{n=1}^{\infty} \frac{1}{N_n}\frac{\lambda_j}{a_j} e^{-\beta_n^2 t} u_{in}(x) u_{jn}(x')\right] T_0(x')dx', \quad i = 1,2 \quad (15)$$

where

$$\overline{T}(x,t) = \begin{cases} \overline{T_1}(x,t), & \text{for } 0 \leqslant x \leqslant x_1 \\ \overline{T_2}(x,t), & \text{for } x_1 < x \leqslant x_2 \end{cases}$$

and the norm N_n is given by

$$N_n = \frac{\lambda_1}{a_1}\int_0^{x_1} u_{1n}^2(x)dx + \frac{\lambda_2}{a_2}\int_{x_1}^{x_2} u_{2n}^2(x)dx \quad (16)$$

The expression in brackets in Eq. (15) is the Green's function for the homogeneous problem. By replacing t by $(t-\tau)$, we obtain the Green's function G_{ij} for the composite medium for the nonhomogeneous case,

$$G_{ij}(x,t|x',\tau) = \sum_{n=1}^{\infty} e^{-\beta_n^2(t-\tau)} \frac{1}{N_n}\frac{\lambda_j}{a_j} u_{in}(x) u_{jn}(x')$$
$$x_{i-1} < x \leqslant x_i, \quad i = 1,2, \quad x_{j-1} < x' \leqslant x_j, \quad j = 1,2. \quad (17)$$

It represents the response at location x and at time t to an impulse located at x' at time τ. There are an infinite number of discrete eigenvalues β_n and the corresponding eigenfunctions u_{in}. The eigenvalues are ordered by magnitude $\beta_1 < \beta_2 < \cdots < \beta_n < \cdots$. For more details, see Refs. 15 to 17.

Finally, the complete temperature distribution in the sample can be calculated by the resulting formula, where $T_1(x,t)$ stands for the temperature in the inner layer and $T_2(x,t)$ for that in the outer layer;

$$T_i(x,t) = \sum_{j=1}^{2} \left\{ \int_{x_{j-1}}^{x_j} G_{ij}(x,t|x',\tau)\big|_{\tau=0} T_0(x')dx' + \int_0^t \int_{x_{j-1}}^{x_j} G_{ij}(x,t,|x',\tau)\frac{a_j}{\lambda_j} q(x,'\tau)dx'd\tau \right\} \quad (18)$$

$$\text{for } x_{i-1} < x \leqslant x_i, \quad i = 1, 2$$

We assume that the initial temperature in the sample at $t=0$ is constant, $T_0(x) = T_0$, as can be expected for the experimental configuration. Then, using the substitution,

$$\Delta T_i(x,t) = T_i(x,t) - T_0 \quad (19)$$

for the temperature rise ΔT_i, the derivation of the solution is very similar, but in the resulting expression, the first integral of Eq. (18) vanishes. Remember that the constant heat source (Eq. (5)) is limited to a thin heater, in the one-dimensional model to a short interval, and the solution becomes

$$\Delta T_i(x,t) = q_0 \frac{a_1}{\lambda_1} \int_0^t \int_0^d G_{i1}(x,t|x',\tau)dx'd\tau \quad \text{for } x_{i-1} < x \leqslant x_i, \quad i = 1, 2. \quad (20)$$

The transient signal, measured, i.e., the temperature rise in the source plane, is calculated from Eq. (20) and the derived Green's function (Eq. (17)) at $x=0$ to be

$$\Delta T_1(0,t) = q_0 \sum_{n=1}^{\infty} \frac{\sqrt{a_1}}{\beta_n^3 N_n} \left[1 - e^{-\beta_n^2 t}\right] \sin\left(\frac{\beta_n}{\sqrt{a_1}} d\right). \quad (21)$$

Adding the initial temperature T_0 at $t=0$ corresponding to Eq. (19), we get the temperature signal $T_1(0,t)$, the solution of the forward problem. The summands of the infinite series in Eq. (21) fast converge to zero for growing eigenvalues.

2.3. Inverse Problem

The objective is to find the thermal conductivity and thermal diffusivity values of the two-layered composite under test that are consistent with the experimental measuring signal. The underlying mathematical model relating the experimental setup and the thermal properties of the sample with the measuring signal is generally given by Eqs. (4) to (6). The explicit form, Eq. (21), has been derived in the last section. This analytic solution has the advantage of requiring very little computing time compared with numerical solution methods such as finite-element or finite-difference methods. On the other hand, the experimental configurations for which analytic expressions can be derived, simulating the measuring signal in multi-layered samples are limited to special cases.

Let the vector $\boldsymbol{T}^{sim}(\lambda_1, a_1, \lambda_2, a_2, t)$ be the simulated measuring signal depending on the thermal properties and discrete times $\boldsymbol{t} = (t_1, \ldots, t_s)$, and $\boldsymbol{T}^{mes}(\boldsymbol{t})$ the vector of the measuring signal. The related inverse problem of parameter identification is formulated as an output-least-squares problem,

$$\left\| \boldsymbol{T}^{sim}(\lambda_1, a_1, \lambda_2, a_2, \boldsymbol{t}) - \boldsymbol{T}^{mes}(\boldsymbol{t}) \right\|_2^2 = \min!$$

based on the repeated solving of the forward problem, in conjunction with a minimization strategy. The subscript indicates the l_2-norm. It is solved by the Levenberg–Marquardt method [18] as published by the program library of the International Mathematical Subroutine Library (IMSL). The algorithm combines the Gauss–Newton method with the gradient method which is well suited for handling ill-conditioned problems. This procedure was also successfully applied to homogeneous and multilayered problems, solving the forward problem by a finite-element method [8,9]. For layered composites, the subsequent determination of the parameters, beginning with the inner layer, strongly improves the condition of the problem. In a first step, a particular initial interval of the signal is used for the parameter identification of the inner layer and, in a following step, an adequately longer interval is used for the second layer.

3. NUMERICAL EXPERIMENTS

Now, we test the proposed technique by reconstructing thermal transport properties of a layered sample obtained from simulated data. The geometrical dimensions and the thermal properties of the sample are chosen as follows:

The thickness of the heater $2d = 20\mu$m, the thickness of the inner layer $d_1 = 20$ mm, the thickness of the outer layer $d_2 = 20$ mm, thermal properties of the inner layer $\lambda_1 = 1.5\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ and $a_1 = 1.0\,\mathrm{mm^2 \cdot s^{-1}}$ and of the outer layer $\lambda_2 = 0.5\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ and $a_2 = 0.5\,\mathrm{mm^2 \cdot s^{-1}}$. In comparison to pulse heating the sample size is larger.

First, the matrix of coefficients is calculated as given in Eq. (13). From the requirement that the determinant of the coefficients must vanish, we determine the eigenvalues of the corresponding eigenvalue problem (Eq. (10)). Fig. 2 shows the determinant as a function of β where the zeros are the wanted eigenvalues. A precise calculation was achieved using a subroutine of the IMSL library for the determination of zeros of transcendental functions and furnishes an arbitrary number of eigenvalues β_n. The current number r needed depends on the convergence properties of the series in Eq. (21). In our example, all summands for $\beta_i \geqslant 1.5\,\mathrm{s}^{-1/2}$ are negligibly small. Equation (10) has $r = 21$ eigenvalues satisfying the condition $\beta_i < 1.5\,\mathrm{s}^{-1/2}$, as listed in Table I.

The corresponding measuring signal calculated by Eq. (21) as well as the signal for a homogenous sample (λ_1 and a_1), are shown in Fig. 3. In the first interval $[0, t_z]$, only the thermal properties of the inner core govern the temperature rise. Therefore, the curves of the two-layered and

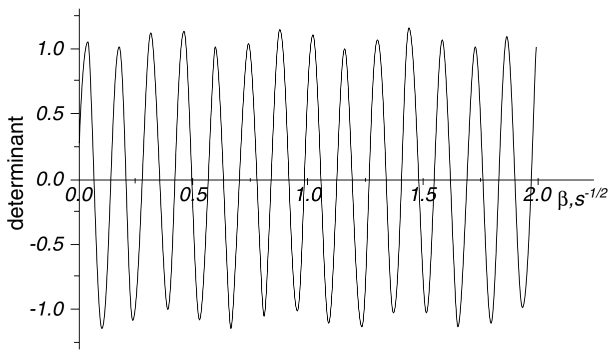

Fig. 2. Determinant of the coefficients given in Eq. (13) versus β.

Table I. Eigenvalues of Eq. (10) Smaller than 1.5 [in units of $\mathrm{s}^{-1/2}$]

0.0692	0.5640	1.0562
0.1424	0.6322	1.1279
0.2103	0.7055	1.1955
0.2813	0.7743	1.2683
0.3535	0.8440	1.3384
0.4210	0.9170	1.4069
0.4935	0.9847	1.4802

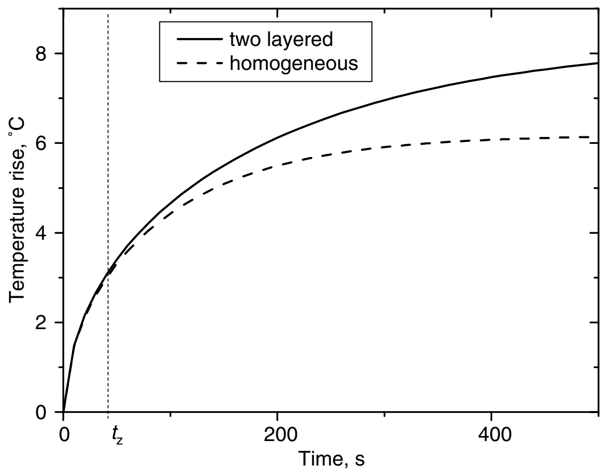

Fig. 3. Calculated temperature rise for a layered sample and a corresponding homogeneous sample (a_1, λ_1).

the homogeneous sample coincide. For $t > t_z$ the temperature rise is determined by the thermal conductivity and thermal diffusivity of both the inner core and the outer layer, as well as at later times by also the surroundings.

As expected from earlier investigations [9] using the finite-element method for the forward problem of the hot-strip technique, the inverse problem of the simultaneous identification of the four properties $\lambda_1, a_1, \lambda_2$, and a_2 is highly ill-posed. To improve the condition of the problem, the thermal properties have to be determined one after another, starting with the inner layer. As a first step, the initial interval $[0, t_z]$ of the signal is selected to identify the thermal properties of the inner layer. In a subsequent step, the determination of the properties of the outer layer is carried out within the remaining interval ($t > t_z$) while keeping the results of the first step fixed. As shown in Ref. 9, the total measurement period has to be sufficiently long so that the signal contains information about the sample surface. Otherwise, the thermal properties cannot be uniquely determined and the iteration may lead to a local minimum. For the analytic solution (Eq. (21)) adiabatic boundary conditions are assumed. However, other conditions can be analogously included, assuming that in case of boundary conditions of the third kind, the surface conductance is known. The same is valid for imperfect contact at the interface.

The measurement uncertainty for the inner layer is the same as for a homogenous material, but in comparison to transient hot-strip (THS) techniques [19], a portion of the model error is strongly decreased.

Numerical experiments showed that the accuracy for thermal properties of the outer layer is a little smaller, especially for the thermal diffusivity.

Theoretically, the method can be extended to more than two layers. Nevertheless, a degradation of the condition of the inverse problem and the existence of local minima can be expected, resulting in a larger uncertainty of the results.

4. SUMMARY

Transient methods are widely used to determine the thermal properties of some materials, but almost always for homogeneous media. For the situation of layered composites a new fast identification algorithm is presented which is based on an analytic solution of the forward problem and a numerical least-squares solver. For the measuring configuration, a plane source is favored because a one-dimensional treatment is possible in this case. The method is designed for simultaneous determination of the four properties, viz., thermal diffusivity and thermal conductivity of the two layers.

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